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AN ERROR ELIMINATION ALGORITHM OF A POLE-ASSIGNMENT SELF-TUNER FOR STEP REFERENCE INPUTS

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This paper proposes a modified version of pole assignment self-tuners for linear time-invariant systems subjected to unknown constant disturbances. It is shown that this version also has the local convergence property established by Goodwin and Sin(1981, 1984) which was developed for linear systems with no disturbances. Utilizing this property, an error elimination algorithm is developed for step references. The performance of this algorithm is demonstrated through a simulation and the result is compared with that of a conventional pole assignment self-tuner using an additional integral action.

Key Words: Pole-Assignment Self Tuner, Non-Minimum Phase System, Local Convergence, Calculation Burden, Transient Performance

1. INTRODUCTION

Closed-loop pole assignment self-tuners (Allidina and Hughes, 1983; Åström, 1980; Eun and Cho; Goodwin and Sin, 1981, 1984; Ortega and Kelly, 1984; Wellstead, Prager and Zanker, 1979) have been utilized to effectively handle non-minimum phase systems since these algorithms do not cancel any of system unstable zeros, while modifying only the system poles. Goodwin and Sin(1981, 984) have established the local convergence of these self-tuners for discrete, deterministic, linear time-invariant systems which have no disturbances. The convergence means that the system inputs and outputs remain bounded for all time and that the closed loop poles are effectively assigned in the limit for a given desired trajectory. This paper proposes a modified version of pole assignment self-tuners for the systems subjected to unknown constant disturbances (see Fig. 1) and proves its local convergence property. Utilizing this property, an error elimination algorithm is developed for step reference inputs [see Fig. 2(b)]. It is shown that, as an alternative of the conventional pole

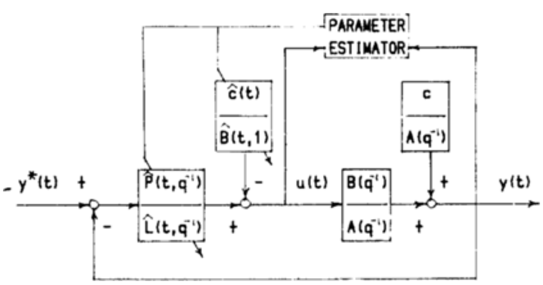
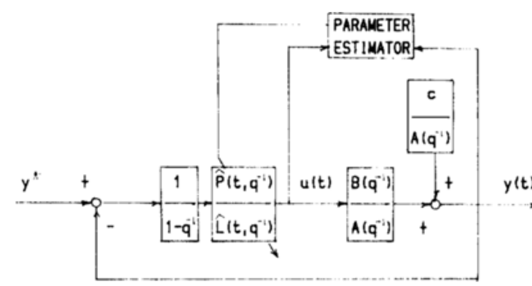
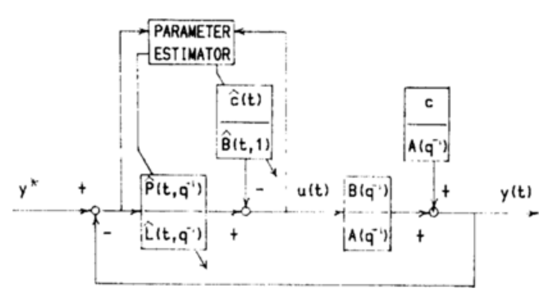


Fig. 1 Block diagram of the modified pole assignment self-tuner



(a) Additional integral action



(b) The proposed algorithm

Fig. 2 Block diagram comparison of the pole assignment self-tuners to eliminate the system steady-state error

assignment self-tuners using an additional integral action [see Fig. 2(a)], this algorithm can eliminate steady-state errors without increasing the minimal order of the controller and calculation burden to obtain the controller parameters. The transient performance of this algorithm is demonstrated through a simulation study and the result is compared with that of the additional integral action.

2. FORMULATION OF THE ERROR ELIMINATION ALGORITHM

Consider a linear time-invariant system subjected to an

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unknown constant disturbance. The system equation is described by:

$$A(q^{-1})y(t)=B(q^{-1})u(t)+c \tag{1}$$

where q^{-1} denotes the backward operator and c represents an unknown constant disturbance. The polynomials in (1) are defined as

$$A(q^{-1})=1+a_1q^{-1}+\cdots+a_nq^{-n} \tag{2}$$
$$B(q^{-1})=b_1q^{-1}+\cdots+b_mq^{-m} \tag{3}$$

The parameter estimation algorithm is given by

$$\hat\theta(t)=\hat\theta(t-1)+\frac{Q(t-2)\phi(t-1)}{1+\phi(t-1)^TQ(t-2)\phi(t-1)}e(t) \tag{4}$$

$$Q(t)=Q(t-1)-\frac{Q(t-1)\phi(t)\phi(t)^TQ(t-1)}{1+\phi(t)^TQ(t-1)\phi(t)} \tag{5}$$

with $Q(0)=\varepsilon I$, ε is any positive real number, where

$$e(t)\triangleq y(t)-\phi(t-1)^T\hat\theta(t-1) \tag{6}$$
$$\hat\theta(t)\triangleq[-\hat a_1(t),\cdots,-\hat a_r(t),\ \hat b_1(t),\cdots,\ \hat b_r(t),\ \hat c(t)]^T \tag{7}$$
$$\phi(t)\triangleq[y(t),\ y(t-1),\cdots,y(t-r),\ u(t-1),\cdots,\ u(t-r),\ 1]^T \tag{8}$$
$$\hat A(t,\ q^{-1})\triangleq 1+\hat a_1(t)q^{-1}+\cdots+\hat a_r(t)q^{-r} \tag{9}$$
$$\hat B(t,\ q^{-1})\triangleq\hat b_1(t)q^{-1}+\cdots+\hat b_r(t)q^{-r} \tag{10}$$

and $r\triangleq\max\{n,\ m\}$

The modified pole assignment control law is proposed as follows:

$$\hat L(t,\ q^{-1})u(t)=\hat P(t,\ q^{-1})[y^*(t)-y(t)]-\frac{1}{\hat B(t,\ 1)}\hat L(t,\ q^{-1})\hat c(t) \tag{11}$$

$$\hat A(t,\ q^{-1})\hat L(t,\ q^{-1})+\hat B(t,\ q^{-1})\hat P(t,\ q^{-1})=A^*(q^{-1}) \tag{12}$$

where $y^*(t)$: the desired output trajectory

$A^*(q^{-1})$: the desired closed-loop characteristic polynomial

$$\hat L(t,\ q^{-1})\triangleq 1+\hat l_1(t)q^{-1}+\cdots+\hat l_{r-1}(t)\ q^{-r+1} \tag{13}$$
$$\hat P(t,\ q^{-1})\triangleq\hat p_0(t)+\hat p_1(t)q^{-1}+\cdots+\hat P_{r-1}(t)\ q^{-r+1} \tag{14}$$

2.1 The Local Convergence of the Modified Pole Assignment Control Law

To implement the control law proposed in (11), $\hat B(t,\ 1)$ should not be zero for all time. This requires the assumption that $B(q^{-1})$ has no factor of $(1-q^{-1})$, i.e., $B(1)\neq 0$. Then, we can choose an initial estimation value of $\hat B(t,\ q^{-1})$ such that $\hat B(t,\ 1)$ is not zero for all time. The required assumptions including the above can be summarized as follows:

(1) $r=\max\{n,\ m\}$ is known.
(2) $A(q^{-1})$ and $B(q^{-1})$ are relatively prime.
(3) $A^*(q^{-1})$ is an arbitrary stable monic polynomial of order $(2r-1)$.
(4) $\{y^*(t)\}$ is an arbitrary bounded set point sequence.
(5) $B(q^{-1})$ has no factor of $(1-q^{-1})$.

Under these assumptions the algorithms (4) through (14) lead to

(1) $\{u(t)\}$ bounded
(2) $\{y(t)\}$ bounded
(3) the closed loop characteristic polynomial tends close to $A^*(q^{-1})$ in the sense that

$$\lim_{t\to\infty}[A^*(q^{-1})y(t)-G(t-1,\ q^{-1})\ y^*(t)]=0 \tag{15}$$

where

$$G(t-1,\ q^{-1})\triangleq\sum_{j=1}^{r}\ \hat b_j(t-1)\sum_{k=1}^{r-1}\hat pk(t-1)q^{-j-k}.$$

[Proof] By following the Goodwin and Sin's methodology(1981, 1984) in a straightforward manner, we can easily prove this modified version. Hence, except the additional parts distinct from their procedures, we will omit the detailed discussions and utilize the time-varying operators without newly defining them. The filtered values $w(t)$ and $z(t)$ of the reference input are modified as follows:

$$w(t)=\hat A\cdot\hat P\ y^*-\left[\hat P+\hat A\cdot\left(\frac{1}{\hat B(t,\ 1)}\hat L\right)\right]\hat c \tag{16}$$
$$z(t)=\hat B\cdot\hat P\ y^* \tag{17}$$

After several steps of arithmetic rearrangements, we can obtain the following MIMO time-varying discrete dynamic equation.

$$\begin{bmatrix} A^*+[\hat A\cdot\hat L-\hat A\hat L]+[\hat P\cdot\hat B-\hat P\hat B] & [\hat A\cdot\hat P-\hat A\hat P]-[\hat P\cdot\hat A-\hat P\hat A] \\ [\hat B\cdot\hat L-\hat B\hat L]-[\hat L\cdot\hat B-\hat L\hat B] & A^*+[\hat B\cdot\hat P-\hat B\hat P]+[\hat L\cdot\hat A-\hat L\hat A] \end{bmatrix}\begin{bmatrix} u(t) \\ y(t)\end{bmatrix}=\begin{bmatrix} w(t)-\hat P\ e(t)-\hat P[\hat c(t-1)-\hat c(t)] \\ z(t)+\hat Le(t)+[\hat L\hat c(t-1)-\hat B\cdot\left(\frac{1}{\hat B(t,\ 1)}\hat L\right)\hat c(t)]\end{bmatrix} \tag{18}$$

The dynamic equation in the above is very much similar to that derived by Goodwin and Sin(1981, 1984) except that the terms $\hat P[\hat c(t-1)-\hat c(t)]$ and $[\hat L\ \hat c(t-1)-\hat B\cdot\left(\frac{1}{\hat B(t,\ 1)}\hat L\right)\hat c(t)]$ are added to the left side of the equation. From the convergence lemma(1981, 1984) of the estimation algorithm it is shown that the coefficients of $\hat A(t,\ q^{-1})$ and $\hat B(t,\ q^{-1})$ and $\hat c(t)$ converge into certain finite values (not necessary to converge into their true values). From the pole assignment equation the convergence of the estimated parameters leads to the convergence of the controller parameters in $\hat P(t,\ q^{-1})$ and $\hat L(t,\ q^{-1})$. Utilizing these results, it follows that the above two terms different from Goodwin and Sin's(1981, 1984) approach zero as t tends to infinity. Utilizing the procedures suggested by Goodwin and Sin(1981, 1984), we can prove the boundedness of $\{u(t)\}$ and $\{y(t)\}$ and establish the final part of this theorem given in (15).

2.2 The Error Elimination Algorithm for Step References

Consider the system described in (1) through (14). Provided that the reference input $y^*(t)$ is a finite constant value, the control algorithm (11) can make the steady state error of the closed-loop system approach zero under the redefinition of the $\phi(t)$ given in (8).

$$\phi(t) \triangleq [y(t)-y^*, \cdots, \ y(t-r)-y^*, \ u(t-1), \cdots, \ u(t-r), \ 1]^T \quad (19)$$

[Proof] Utilizing the convergence property of the modified pole assignment self-tuner, this corollary can be proved in a straightforward manner by rewriting (11) and by subtracting $A(q^{-1})y^*$ from both sides of the system Eq. (1).

$$\hat{L}(t, \ q^{-1})u(t) = -\hat{P}(t, \ q^{-1})\tilde{y}(t) - \frac{1}{\hat{B}(t, \ 1)} \hat{L}(t, \ q^{-1})\hat{c}(t) \quad (20)$$

$$A(q^{-1})(y(t)-y^*) = B(q^{-1})u(t) + c - A(q^{-1})y^* \quad (21)$$

where $\tilde{y}(t) \triangleq y(t) - y^*$. The last equation can be rewritten as

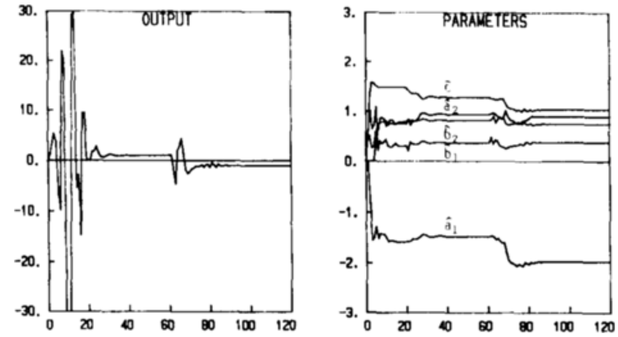
$$A(q^{-1})\tilde{y}(t) = B(q^{-1})u(t) + c^1 \quad (22)$$

by defining $c^1 = c - A(1)y^*$. (23)

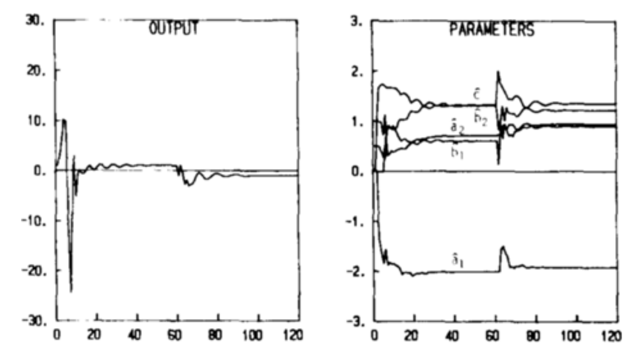
The resulting system described in (22) can be interpreted as a system whose input, output and offset are $u(t)$, $\tilde{y}(t)$, and c^1, respectively, while the reference input value for $\tilde{y}(t)$ is always zero. It is noted that y^* is included in the new disturbance term defined in (23). Considering the convergence property described in (15) and $\tilde{y}^*(t) = 0$, it can be shown that $\tilde{y}(t)$ approaches zero, i.e., $y(t)$ converges into y^* as time tends to infinity.

3. DISCUSSIONS AND CONCLUSION

We have proposed the modified version of pole assignment self-tuners as shown in Fig. 1, and established its local convergence property. Utilizing this, an error elimination algorithm is developed for constant reference inputs. Fig. 2 describes structures of the pole assignment self-tuners utilizing (a) additional integral action and (b) the proposed error elimination algorithm. The structural difference between the two is that the proposed algorithm posseses the parameter estimator which utilizes control inputs and system errors whereas the estimator of the conventional algorithm utilizes control inputs and system outputs. The performances of these self-tuners are investigated by a series of simulation works. During the simulation instead of the recursive least square algorithm[Eq. (4) through (10)] the recursive projection algorithm (Eun and Cho) is utilized as a parameter estimation algorithm without any change of the above stability results (Goodwin and Sin, 1981, 1984). The simulation conditions and its numerical results are precisely described in Table 1. The reference input used for this simulation is given by $y^*(t) = 1$ for $0 \le t < 60$ and -1 for $60 \le t \le 120$.



(a) Additional integral action



(b) The proposed algorithm

Fig. 3 Simulation results
($y^*(t) = 1$ for $0 \le t < 60$ and $y^*(t) = -1$ for $60 \le t \le 120$)

As shown in Fig. 3, both the integral action and the proposed algorithm can make the system steady-state error approach zero, although the system parameters do not converge their true values. For the discussion about the transient performances we examine the response for the second step (at $t = 60$) because the first step responses are severely dependent on the initial parameter estimates. From these responses it can be seen that the proposed algorithm takes slightly longer settling time to reach the steady state. This is due to the fact that parameter adaptation time is additionally required for the algorithm because the variation of reference inputs is regarded as a variation of a system disturbance parameter to be estimated. Therefore, it can be concluded that the performance of the proposed error elimination algorithm is very dependent upon the adaptation speed of the parameters, i.e., the capability of the parameter estimation algorithm. In view of calculation burden the proposed algorithm is comparable with the additional integral action, because the additional integral action increases the minimal order of the controller while the proposed algorithm increases the number of parameters to be estimated. But when a system inherently includes

Table 1 The numerical data for the example studies($y^*(t) = 1$ for $0 \le t < 60$ and $y^*(t) = -1$ for $60 \le t \le 120$)

	t	$y(t)$	$\hat{a}_1$	$\hat{a}_2$	$\hat{b}_1$	$\hat{b}_2$	$\hat{c}$
True values			−2.	0.96	0.5	1.	1.
Adding integral action	0 60 120	0. 0.99946 −1.00000	0. −1.47796 −1.96793	0.5 0.93586 0.74518	1. 0.36478 0.37613	0. 0.81424 0.89392	0. 1.27496 1.03559
Using the proposed algorithm	0 60 120	0. 0.98414 −0.99798	0. −2.00359 −1.91446	0.5 0.72193 0.94999	1. 0.61097 0.90310	0. 1.31472 1.22773	0. 1.33181 1.36451

a constant disturbance as most practical systems do, the proposed algorithm reduces the calculation burden because it does not increase the number of parameters to be estimated. These results indicate that for step reference inputs the proposed error elimination algorithm can be a good alternative to the additional integral action.

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